Spin-magnetophonon level splitting in semimagnetic quantum wells

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Spin-magnetophonon level splitting in a quantum well made of a semimagnetic wide gap semiconductor is considered. The semimagnetic semiconductors are characterized by a large effective g factor. The resonance conditions $\hbar\omega_{\mathrm{LO}}=\mu_B gB$ for the spin flip between two Zeeman levels due to the interaction with longitudinal optical phonons can be achieved at sweeping magnetic field B. This condition is studied in quantum wells. It is shown that it leads to a level splitting that is dependent on the electron-phonon coupling strength as well as on the spin-orbit interaction in this structure. We treat in detail the Rashba model for the spin-orbit interaction, assuming that the quantum well lacks inversion symmetry and briefly discuss other models. The resonant transmission and the reflection of light by the well are suggested as suitable experimental probes of the level splitting.

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I. INTRODUCTION

The resonance coupling of Landau levels with longitudinal optical phonons (magnetophonon resonance) was theoretically predicted in Ref. 1 in the magnetoresistance investigation. The resonance takes place every time the optical-phonon frequency is the cyclotron frequency of an electron times some small integer. Thus, a possibility has been pointed out for an internal resonance in solids. This phenomenon (since its prediction) has been observed in many experiments—see for instance the review.[2]

A possibility of the spin-flip transitions of electrons interacting with the optical phonons between the Landau levels of opposite spin orientations that may be called spin-magnetophonon resonance (SMPR) was indicated and discussed in a number of papers (see Refs. 3–6). The purpose of this paper is to discuss the peculiarities of SMPR in semimagnetic semiconductors where, due to large effective g factors, the corresponding interlevel spacing may be particularly large and, therefore, SMPR is well pronounced. The condition for the spin resonance has the form

$$g\mu_{\mathrm{B}}B=\hbar\omega_{\mathrm{LO}}. \tag{1}$$

Here μ_{B} is the Bohr magneton, g is the carrier effective g factor, while B is the external magnetic field.

Many remarkable magneto-optical properties of wide gap semimagnetic semiconductors, such as giant exchange splitting of the free exciton,[7] giant Faraday effect,[7–9] etc. are determined by a large splitting of conduction and valence bands in the magnetic field. This is a consequence of the exchange interaction of band carriers with the electrons of the half-filled d shell of the Mn ions. In the present paper we will treat the compound $Cd_{1-x}Mn_xTe$ as an example, where the width of the gap between the top of valence band and the bottom of conduction band in the absence of the magnetic field is given by $E_g=1.595+1.592x$ eV.

In the presence of an external magnetic field, the Mn-ion spins are aligned along the magnetic field. Through the exchange of the Heisenberg type, these spins interact with the spins of the band carriers. Eventually, in the mean-field model, the band carrier dynamics can be described by incorporating the exchange interaction only into the enhanced g factor.

There are two competing mechanisms determining the sign and value of the exchange constant (and of the g factor).[10–12] The first mechanism originating from direct exchange interaction between the band and d electrons is relatively weak and ferromagnetic. The second one is due to the hybridization of d orbitals and band states. The latter turns out to be antiferromagnetic and is negligible for the conduction band, while for the valence band it determines the exchange constant.

The resonance coupling of Landau levels with optical phonons also manifests itself in a different way, although the underlying physics is the same. It leads to magneto-optical anomalies both in bulk[13] and in two-dimensionally confined systems.[14,15] Primary concern of Refs. 14 and 15 was the investigation of magneto-optical anomalies of optical phenomena in conventional GaAs based heterostructures. It was shown that magneto-optical anomalies in two dimensions provide a powerful tool for the electron-phonon coupling investigation in these structures. It was found that under the resonance condition with respect to electron-phonon interaction, the relevant cyclotron peak splits into a doublet. This effect leads to anomalies in optical absorption and reflection (as well as in other optical effects such as, for instance, Raman scattering).

In this paper we investigate the effect associated with SMPR, i. e., the magnetophonon resonances due to the spin flips. These electron-phonon resonance conditions can occur both for the valence and conduction electrons. The exchange constants for the conduction and valence electrons turn out to be different.[16] Though the resonance condition leading to the level splitting occurs first in the valence band, we will show that the splitting itself is smaller for the valence-band states than for the states in the conduction band.

In Sec. II we consider the level splitting as a formal quantum-mechanical problem. This phenomenon can be understood in terms of degeneracy lifting of two degenerate states. The energy degeneracy of an electron in a state 2 and an electron in a state 1 plus an optical phonon (see Fig. 1 where the states $0,0+$ correspond to state 1 and state 2, re-

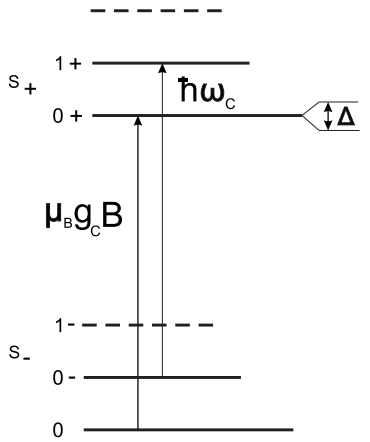

FIG. 1. Level splitting in the conduction band. Only the electron levels in resonance with phonons are shown.

spectively) is lifted by the electron-phonon interaction. We will obtain an expression for the level splitting without specifying the states involved in the relevant transitions. In Sec. III we determine the states and the energy levels of the conduction electrons taking into account the spin-orbit interaction in the Rashba model. This allows us to express the level splitting explicitly. We give the required estimations at the end of Sec. III. As a possible experimental probe of this splitting phenomenon, we propose the resonant reflection (transmission) of the light by a quantum well in the Faraday configuration. We consider the wave reflection (transmission) due to direct interband transitions; therefore, in Sec. IV we give explicit expressions for the wave functions and energies of the valence-band states. In Sec. V these wave functions are used to determine the reflection and transmission coefficients of the light exciting interband transitions in the quantum well. In Sec. VI we discuss applicability of the perturbation theory for the solution of our problem. We present conclusive remarks in Sec. VII.

II. LEVEL SPLITTING

We begin with the treatment of a formal problem; we will consider two states 1 and 2 and find the self-energy of an electron in the state 2 due to the interaction of the electron with optical phonons. Suppose the energy of the state under consideration ε_2 is close to $\varepsilon_1+\hbar\omega_{\mathrm{LO}}$ (i.e., the electron state 2 and the electron state 1 plus the optical phonon with frequency ω_{LO} are degenerate). This allows us to put aside all other possible electron states.

Generally, a single-quantum well brings about phonon (vibrational) modes. There could be three types of phonons associated with a quantum well;[17] phonons not penetrating into the quantum well, phonons peaking at the interface and decaying both in the well and in the barriers (interface phonons), and phonons confined to the well. The phonon Green's function in the Matsubara technique can be written as

$$D(\mathbf{r}_\perp,z,z',i\omega_k)=-\sum_{\alpha\mathbf{q}_\perp}|C_\alpha|^2\left[\frac{e^{i\mathbf{q}_\perp\mathbf{r}_\perp}\eta_\alpha(z)\eta_\alpha^*(z')}{i\omega_k+\hbar\omega_{\mathrm{LO}}}-\frac{e^{-i\mathbf{q}_\perp\mathbf{r}_\perp}\eta_\alpha^*(z)\eta_\alpha(z')}{i\omega_k-\hbar\omega_{\mathrm{LO}}}\right], \tag{2}$$

where $\eta_\alpha(z)$ describes the spatial distribution of the phonon α branch in the direction perpendicular to the well plane (z axis), $\omega_k=2\pi kT$ ($k=0,\pm1\ldots$) are the Matsubara boson frequencies, and $|C_\alpha|^2$ is the electron-phonon coupling strength. T is the temperature; we will use for it the energy units setting $k_{\mathrm{B}}=1$.

The electron-phonon interaction with longitudinal optical phonons can be treated in the bulk Fröhlich[18] model. According to the model $\eta_\alpha(z)\to e^{iq_zz}$, $|C_\alpha|^2\to 2\pi e^2\hbar\omega_{\mathrm{LO}}/q^2\epsilon^*$. This approximation in relatively wide wells can be justified, noting that the interaction with the interface phonons in this case can be neglected; interaction with the confined phonons qualitatively leads to the same result. Therefore, further on we will work in the Fröhlich approximation. We consider the dispersionless optical phonons, with ω_{LO} being their frequency and

$$\frac{1}{\epsilon^*}=\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}, \tag{3}$$

where $\epsilon_\infty(\epsilon_0)$ is the high-frequency (static) limit of the dielectric susceptibility.

The electron self-energy in the first approximation of the perturbation theory with respect to the electron-phonon interaction can be written as [see the diagram (a) in Fig. 6]

$$\Sigma_2(i\varepsilon_n)=-T\frac{2\pi\omega_{\mathrm{LO}}e^2\hbar}{\epsilon^*}\sum_k\frac{F_{21}}{i(\varepsilon_n-\omega_k)-\varepsilon_1+\mu}\frac{2\hbar\omega_{\mathrm{LO}}}{\omega_k^2+(\hbar\omega_{\mathrm{LO}})^2} \tag{4}$$

where $\varepsilon_n=\pi(2n+1)T$ and

$$F_{21}=\int\frac{d^3q}{(2\pi)^3}\frac{|\langle 2|e^{i\mathbf{q}\mathbf{r}}|1\rangle|^2}{q^2}. \tag{5}$$

For the self-energy, we get

$$\Sigma_2(i\varepsilon_n)=-\frac{2\pi\omega_{\mathrm{LO}}e^2\hbar}{\epsilon^*}F_{21}\left[\frac{n_F(\varepsilon_1)-n_B(\omega_{\mathrm{LO}})-1}{i\varepsilon_n-(\varepsilon_1-\mu)-\hbar\omega_{\mathrm{LO}}}-\frac{n_F(\varepsilon_1)+n_B(\omega_{\mathrm{LO}})}{i\varepsilon_n-(\varepsilon_1-\mu)+\hbar\omega_{\mathrm{LO}}}\right]. \tag{6}$$

Restricting ourselves with the low-temperature case $T\ll\hbar\omega_{\mathrm{LO}}$ and assuming that the state ε_1 is empty, we get

$$\Sigma_2(i\varepsilon_n)=\frac{\Delta^2/4}{i\varepsilon_n-(\varepsilon_1-\mu)-\hbar\omega_{\mathrm{LO}}}, \tag{7}$$

where

$$\Delta^2=\frac{8\pi\omega_{\mathrm{LO}}e^2\hbar}{\epsilon^*}F_{21}. \tag{8}$$

For the electron Green's function, we get this self-energy

$$G_2(i\varepsilon_n) = \frac{1}{i\varepsilon_n - \varepsilon_2 + \mu - (\Delta/2)^2/(i\varepsilon_n - \varepsilon_1 - \hbar\omega_{\mathrm{LO}} + \mu)}. \tag{9}$$

For the retarded Green's function, we have

$$G_2^R(\varepsilon) = \frac{\varepsilon - \varepsilon_1 - \hbar\omega_{\mathrm{LO}} + \mu}{(\varepsilon - \varepsilon_+ + \mu + i0)(\varepsilon - \varepsilon_- + \mu + i0)}, \tag{10}$$

where

$$\varepsilon_\pm = \frac{\varepsilon_2 + \varepsilon_1 + \hbar\omega_{\mathrm{LO}}}{2} \pm \sqrt{[(\varepsilon_2 - \varepsilon_1 - \hbar\omega_{\mathrm{LO}})/2]^2 + (\Delta/2)^2}. \tag{11}$$

As seen from Eq. (10) we have gotten two poles of the Green's function; the level ε_2 is split into a doublet with the energies $\varepsilon_\pm$, the spacing between the poles being equal Δ. The splitting can be expressed through the parameter α describing the effective-mass polaron shift,

$$\Delta^2 = 16\pi\alpha l_{\mathrm{LO}}(\hbar\omega_{\mathrm{LO}})^2 \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{|\langle 2|e^{i\mathbf{q}\mathbf{r}}|1\rangle|^2}{q^2},$$

$$\alpha^2 = \frac{m_c e^4/2(\hbar\epsilon^*)^2}{\hbar\omega_{\mathrm{LO}}}. \tag{12}$$

Here we introduced the length $l_{\mathrm{LO}} = \sqrt{\hbar/2m_c\omega_{\mathrm{LO}}}$, where m_c is the electron effective mass. The parameter α for materials with a relatively weak polarity is small. For instance, $\alpha = 0.39$ for CdTe with partly ionic bonding. Suppose now that we can achieve the resonant condition, $\varepsilon_2 - \varepsilon_1 = \hbar\omega_{\mathrm{LO}}$ changing the interlevel spacing $\varepsilon_2 - \varepsilon_1$. If the state 2 and state 1 are the spin-up and spin-down states, respectively, then the resonant condition can be reached by adjusting external magnetic field. Since the level splitting is proportional to a matrix element $\langle 2|e^{i\mathbf{q}\mathbf{r}}|1\rangle$, we see that the phonons can lead to spin flips only provided that the state 2 and state 1 are not the eigenfunctions of spin operators $\mathbf{s}^2$ and s_z. For this reason, we must include into the Hamiltonian the spin-orbit interaction. We consider the spin-orbit interaction in the Rashba model,[19]

$$H_R = \frac{\alpha_R}{\hbar}[\sigma\mathbf{p}]\mathbf{n}. \tag{13}$$

Here $\mathbf{n}$ is a unit vector perpendicular to the quantum well plane. This interaction is due to the structure inversion asymmetry. Parameter α_R is of the order of 10^{-9} eV cm.

There could be another spin-orbit interaction term that is due to the bulk inversion asymmetry. The corresponding three-dimensional (3D) spin-orbit Dresselhaus Hamiltonian[20] in the crystal principal axes reads,

$$H_D = \delta(\sigma\mathbf{P}). \tag{14}$$

Here $\hbar^3 P_x = p_y p_x p_y - p_z p_x p_z$ and other components of $\mathbf{P}$ can be obtained by cyclic permutations. In a two-dimensional (2D) case, this Hamiltonian takes the form (we omit the terms cubic in p)

$$H_D = \frac{\alpha_D}{\hbar}(\sigma_y p_y - \sigma_x p_x), \tag{15}$$

where $\alpha_D = \delta\langle p_z^2\rangle/\hbar^2$ and $\langle p_z^2\rangle$ is averaged over the transverse motion of the electron. The parameter α_D can be estimated as 10^{-10} eV cm. Being interested only in the possibility of the line splitting in the optical reflection (transmission) experiments with quantum wells, explicit calculations for the spin-orbit interaction of the Rashba form will be presented; since in many semiconductor nanostructures, the Rashba interaction is stronger than the Dresselhaus one. However, it can be shown that the Dresselhaus term in the form (15) does not essentially differ from the Rashba term, so to take into account the Dresselhaus term, one should simply replace the constant α_R with α_D (this will be sufficient for estimations). Indeed, one can show that the Dresselhaus term can be obtained from the last term in Eq. (24) below by simply replacing α_R with α_D and a with $-ia$.

III. DEEP QUANTUM WELL IN TRANSVERSE MAGNETIC FIELD

Let x, y be parallel to the quantum well plane, with z axis being perpendicular to the plane of the well. Further on we will consider the simplest case of an infinitely deep well. We assume that the magnetic field $\mathbf{B}$ is along the z axis (perpendicular to the plane of the well) and choose the gauge $\mathbf{A} = B(0, x, 0)$. In wide gap semiconductors, the conduction and valence bands can be considered separately. In the zinc-blende structures, the conduction-band Hamiltonian near the point Γ_6 is

$$H = H_0 + H_R, \tag{16}$$

$$H_0 = \frac{\hbar^2}{2m_c}\left(-i\nabla + \frac{e}{\hbar c}\mathbf{A}\right)^2 + U + H_Z. \tag{17}$$

Here we use the basis of $Ss_\pm$ (where S is the S-type Bloch amplitude and s_-, s_+ are the two spin functions); U is the confining potential of the quantum well. We write the Zeeman Hamiltonian as

$$H_Z = \frac{1}{2}\mu_B \sigma_z g_c B. \tag{18}$$

Intending to consider the semimagnetic semiconductors $Cd_{1-x}Mn_xTe$, we will incorporate into the Hamiltonian the exchange Heisenberg interaction of the conduction-band electrons with Mn ions,

$$H_{\mathrm{ce}} = -\sum_n J_{\mathrm{ce}}(\mathbf{r} - \mathbf{R}_n)\mathbf{S}_n^{\mathrm{Mn}}\mathbf{s}, \tag{19}$$

where $J_{\mathrm{ce}}(\mathbf{r} - \mathbf{R}_n)$ is the exchange integral of the electron with the Mn ion localized at $\mathbf{R}_n$ site—the sum runs over all the Mn ions. We will use the mean-field approximation inserting the mean value of Mn spin in the z direction $\langle S_z^{\mathrm{Mn}}\rangle$ instead of the corresponding operator and ascribing spin $x\langle S_z^{\mathrm{Mn}}\rangle$ to every crystal site. In this approximation, the exchange Hamiltonian can be rewritten in the form,

$$H_{ce} = -x\langle S_z^{Mn}\rangle N_0 \langle S|J_{ce}(\mathbf{r})|S\rangle s_z \equiv -2\hbar\omega_c V_c s_z, \quad (20)$$

where N_0 is the density of unit cells and $\langle S|J_{ce}(\mathbf{r})|S\rangle$ is the exchange integral (that is assumed to be positive). Here for convenience we factor out the cyclotron frequency $\omega_c = eB/m_c c$. The introduced quantity V_c for the conduction band turns out to be negative and rather large. It can be written as

$$V_c = x\langle S_z^{Mn}\rangle \frac{N_0\langle S|J_{ce}(\mathbf{r})|S\rangle}{2\hbar\omega_c}.$$

The induced Mn-ion spin can be written as

$$\langle S_z^{Mn}\rangle = -\mathcal{B}_S(\zeta), \quad \zeta = \frac{g_{Mn}\mu_B B}{k_B T},$$

where $\mathcal{B}_S(\zeta)$ is the Brillouin function,

$$\mathcal{B}_S(\zeta) = \frac{2S+1}{2}\coth\left(\frac{2S+1}{2}\zeta\right) - \frac{1}{2}\coth\left(\frac{\zeta}{2}\right). \quad (21)$$

For $S=5/2$,

$$\mathcal{B}_{5/2}(\zeta) = \frac{35}{12}\zeta, \quad \zeta \ll 1; \quad \mathcal{B}_{5/2}(\zeta) = \frac{5}{2}, \quad \zeta \gg 1,$$

where $g_{Mn}=2$, μ_B is the Bohr magneton, and $S=5/2$ is the spin of a manganese atom. Therefore, we see that g_c in Eq. (18) must be understood as $g_{zz}-4V_c$. Since $N_0\langle S|J_{ce}(\mathbf{r})|S\rangle = 0.22$ eV (Ref. 16) and $\hbar\omega_c \sim 1$ meV, we get that $g_c \sim 50$.

Eigenfunctions of H_0 as functions of y can be chosen as plane waves $e^{ik_y y}/\sqrt{L_y}$. As functions of z they are the eigenfunctions $\chi_i(z)$ of an infinitely deep one-dimensional well with associated eigenvalues ε_i. Thus, one can rewrite H_0 as

$$H_0 = \varepsilon_i - \frac{\hbar^2}{2m_c}\frac{\partial^2}{\partial x^2} + \frac{m_c\omega_c^2}{2}(x-x_0)^2 + H_Z. \quad (22)$$

Here the position of the center of oscillator $x_0 = -k_y\hbar c/eB$ depends on the quasimomentum along the y direction $\hbar k_y$ (the motion along the y axis is free). The Rashba Hamiltonian in the magnetic field is

$$H_R = \alpha_R\begin{pmatrix} 0 & \partial/\partial x + k_y + x/l_c^2 \\ -\partial/\partial x + k_y + x/l_c^2 & 0 \end{pmatrix}. \quad (23)$$

Here we have introduced the magnetic length $l_c = \sqrt{c\hbar/eB}$. Introducing Bose operators according to $\partial/\partial x = (a - a^\dagger)/(\sqrt{2}l_c)$ and $x - x_0 = l_c(a + a^\dagger)/(\sqrt{2})$, we get

$$H = \varepsilon_i + \hbar\omega_c(a^\dagger a + 1/2) + H_Z + \frac{\sqrt{2}\alpha_R}{l_c}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}. \quad (24)$$

The Rashba term does not change the ground state $\varphi_0(x - x_0)$ and its energy is $\varepsilon_0 = \varepsilon_i + \hbar\omega_c/2 - \mu_B B g_c/2$. Other eigenfunctions of H are

$$\psi_{n+} = \begin{pmatrix} \cos u_n \varphi_n(x-x_0) \\ \sin u_n \varphi_{n+1}(x-x_0) \end{pmatrix}, \quad \psi_{n-} = \begin{pmatrix} -\sin u_n \varphi_n(x-x_0) \\ \cos u_n \varphi_{n+1}(x-x_0) \end{pmatrix}, \quad (25)$$

with the corresponding eigenvalues,

$$\varepsilon_{n\pm} = \varepsilon_i + \hbar\omega_c(n+1) \pm \sqrt{\left(\frac{\hbar\omega_c - \mu_B g_c B}{2}\right)^2 + 2\frac{\alpha_R^2}{l_c^2}(n+1)}, \quad (26)$$

where

$$\tan 2u_n = 2\sqrt{2}\frac{\alpha_R}{l_c}\frac{\sqrt{n+1}}{\mu_B g_c B - \hbar\omega_c}.$$

Here φ_n are the oscillator functions of $x - x_0$,

$$\varphi_n(x-x_0) = \frac{1}{\pi^{1/4}}\frac{1}{\sqrt{2^n n!}}\frac{1}{l_c^{1/2}} \times \exp[-(x-x_0)^2/2l_c^2]H_n[(x-x_0)/l_c], \quad (27)$$

where $H_n(x)$ are the Hermite polynomials. Thus, we arrive at two groups of levels (as is shown schematically in Fig. 1) separated by (large) energy $\mu_B g_c B$. Both groups consist of sublevels that are nearly equidistant (if one neglects the spin-orbit contribution to the energy) and are separated by the cyclotron energy $\hbar\omega_c$. The minimal energy in the first group is ε_0, while in the second group it is $\varepsilon_{0+} = \varepsilon_0 + \mu_B g_c B$. In what follows we restrict ourselves with the phonon induced transitions $0 \to 0+$ and $n- \to (n+1)+$. As we will see below under the realistic conditions, the estimations show that the level splitting due to the electron-phonon interaction turns out to be small as compared to the cyclotron energy; therefore, it is sufficient to consider each pair of states separately. Indeed, for typical magnetic fields of the order of several tesla, the magnetic length $l_c = \sqrt{\hbar c/eB} \sim 10$ nm and the cyclotron energy $\hbar\omega_c = \hbar^2/m_c l_c^2 \sim 10^{-2}$ eV, while the level splitting is of the order $\hbar\Delta\omega \sim 10^{-3}$ eV (see the estimations at the end of Sec. III).

Let us now calculate the matrix element between the ground state 0 of the first group and the 0+ state of the second one,

$$|\langle i,0+,k_y|e^{iq_x x + iq_z z}|i,0,k_y - q_y\rangle|^2 = \sin^2 u_0 |\langle i|e^{iq_z z}|i\rangle|^2 \frac{l_c^2 q_\perp^2}{2}\exp\left(-\frac{l_c^2 q_\perp^2}{2}\right), \quad (28)$$

or taking into account that $u_0 \ll 1$,

$$\Delta_{0+,0}^2 = 4\alpha\frac{l_{LO}}{l_c}(\hbar\omega_{LO})^2 u_0^2 f\left(\frac{l_c}{L}\right), \quad (29)$$

where

$$f\left(\frac{l_c}{L}\right) = l_c\int dz_1 dz_2 \chi_i^2(z_1)\chi_i^2(z_2)\int_0^\infty dq_\perp \frac{q_\perp^2 l_c^2}{2} \times \exp(-l_c^2 q_\perp^2/2 - q_\perp|z_1 - z_2|). \quad (30)$$

Transitions from $n-$ to $(n+1)+$ states are resonant too; for these transitions we have (omitting k_y and $k_y - q_y$),

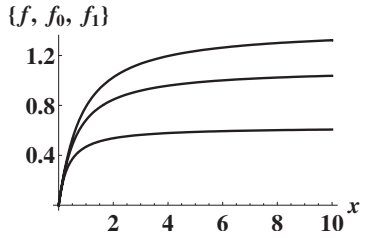

FIG. 2. Functions $f(x)$, $f_0(x)$, and $f_1(x)$ (from the bottom to the top). The second function corresponds to $\Delta^2_{1+,0-}$ and saturates at $x \gg 1$ reaching the value $7\sqrt{2\pi}/16$, while the third one corresponds to the transition $1- \rightarrow 2+$ and also saturates at $x \gg 1$ reaching the value $145\sqrt{2\pi}/256$.

$$|\langle i,(n+1)+|e^{iq_x x+iq_z z}|i,n-\rangle|^2 = 2\left(\frac{\alpha_R}{l_c\mu_B g_c B}\right)^2 |\langle i|e^{iq_z z}|i\rangle|^2 \times \frac{l_c^2 q_\perp^2}{2} e^{-l_c^2 q_\perp^2/2}[L^1_{n+1}(l_c^2 q_\perp^2/2) - L^1_n(l_c^2 q_\perp^2/2)]^2,$$

where $L_n^\alpha(x)$ are the Laguerre polynomials defined in Ref. 21. Here we have taken into account that $\sin u_n \simeq u_n \simeq \sqrt{2(n+1)}\alpha_R/l_c\mu_B B$. Since $L^{\alpha-1}_{n+1}(x)=L^\alpha_{n+1}(x)-L^\alpha_n(x)$, this expression can be simplified,

$$|\langle i,(n+1)+|e^{iq_x x+iq_z z}|i,n-\rangle|^2 = 2\left(\frac{\alpha_R}{l_c\mu_B g_c B}\right)^2 |\langle i|e^{iq_z z}|i\rangle|^2 \frac{l_c^2 q_\perp^2}{2} e^{-l_c^2 q_\perp^2/2}[L_{n+1}(l_c^2 q_\perp^2/2)]^2.$$

We have

$$\Delta^2_{(n+1)+,n-} = 8\alpha\frac{l_{\mathrm{LO}}}{l_c}(\hbar\omega_{\mathrm{LO}})^2\left(\frac{\alpha_R}{l_c\mu_B g_c B}\right)^2 f_n\left(\frac{l_c}{L}\right), \quad (31)$$

$$f_n\left(\frac{l_c}{L}\right) = l_c\int dz_1 dz_2 \chi_i^2(z_1)\chi_i^2(z_2)\int_0^\infty dq_\perp \frac{l_c^2 q_\perp^2}{2} \times \left[L_{n+1}\left(\frac{l_c^2 q_\perp^2}{2}\right)\right]^2 e^{-l_c^2 q_\perp^2/2 - q_\perp|z_1-z_2|}. \quad (32)$$

Here we give an estimation for $\Delta^2_{0+,0}$ assuming that the transverse motion is described by the wave function $\chi_1(z) = \sqrt{2/L}\sin(\pi z/L)$ [see Eq. (29)],

$$\Delta^2_{0+,0} = 8\alpha(\hbar\omega_{\mathrm{LO}})^2\left(\frac{\alpha_R}{l_c\mu_B g_c B}\right)^2\frac{l_{\mathrm{LO}}}{l_c} f\left(\frac{l_c}{L}\right), \quad (33)$$

where $f(x)=\sqrt{2\pi}/4$ for $x \gg 1$ and $f(x)=3x/2$ for $x \ll 1$ (see Fig. 2). Taking into account the CdTe parameters, namely the longitudinal optical-phonon frequency $\omega_{\mathrm{LO}}=3.22\times10^{13}s^{-1}$ (246 K), the susceptibilities $\epsilon_0=10.3$ and $\epsilon_\infty=6.9$, and the effective electron mass $m_c=0.1m_0$ (m_0 is the free-electron mass), we see that the line splitting is $\Delta\omega=\Delta/\hbar \simeq \alpha_R\sqrt{\alpha}/\hbar l_c \sim 10^{11}s^{-1}$. For the Dresselhaus interaction, the line splitting would be α_D/α_R times smaller.

IV. VALENCE BAND

In the structures having zinc-blende symmetry, the valence Γ_8 band is described by the Luttinger Hamiltonian

$$H = H_0 + H(k_z), \quad (34)$$

where we separate the part $H(k_z)$ depending on k_z,

$$H_0 = -\frac{\hbar^2}{2m_0}\left[\left(\gamma_1+\frac{5}{2}\gamma_2\right)(k_x^2+k_y^2) - 2\gamma_2(J_x^2k_x^2+J_y^2k_y^2) - 4\gamma_3(J_xJ_y)(k_xk_y) + 2\frac{e}{c}\kappa\mathbf{J}B\right], \quad (35)$$

$$H(k_z) = -\frac{\hbar^2}{2m_0}\left\{\left(\gamma_1+\frac{5}{2}\gamma_2\right)k_z^2 - 2\gamma_2J_z^2k_z^2 - 4\gamma_3[(J_xJ_z)(k_xk_z) + (J_yJ_z)(k_yk_z)]\right\}. \quad (36)$$

Here $\gamma_1, \gamma_2, \gamma_3, \kappa$ are material parameters and $\mathbf{J}$ is the operator of angular momentum $J=3/2$; the symmetrized products are defined according to

$$\{AB\} = \frac{AB+BA}{2}. \quad (37)$$

We add to the valence-band Hamiltonian the exchange Heisenberg interaction of the valence-band electrons with Mn ions,

$$H_{\mathrm{ve}} = -\sum_n J_{\mathrm{ve}}(\mathbf{r}-\mathbf{R}_n)\mathbf{S}_n^{\mathrm{Mn}}\mathbf{s}, \quad (38)$$

where $J_{\mathrm{ve}}(\mathbf{r}-\mathbf{R}_n)$ is the exchange integral of a valence-band electron with a Mn ion.

The wave function can be written as

$$\Psi = \sum_i F_i(\mathbf{r})u_i(\mathbf{r}), \quad (39)$$

where $u_i(\mathbf{r})$ are the four degenerate states at the top of the valence band,[22]

$$u_{\pm3/2} = \mp\frac{1}{\sqrt{2}}(X \pm iY)s_\pm,$$

$$u_{\pm1/2} = \frac{1}{\sqrt{3}}\left[\mp\frac{1}{\sqrt{2}}(X\pm iY)s_\mp + \sqrt{2}Zs_\pm\right]. \quad (40)$$

It is easily seen that in this basis, the spin operator $s_z=\sigma_z/2$ is also diagonal and is related to the J_z operator by $s_z=J_z/3$; therefore, we can rewrite the exchange Hamiltonian as

$$H_e = -x\langle S_z^{\mathrm{Mn}}\rangle N_0\langle X|J_{\mathrm{ve}}(\mathbf{r})|X\rangle\frac{1}{3}J_z \equiv -2\hbar\omega_{c0}V_vJ_z. \quad (41)$$

Here for convenience we factor out the cyclotron frequency $\omega_{c0}=eB/m_0c$, anticipating its appearance in the following formulae. The introduced quantity V_v for the valence band turns out to be positive and rather large. It can be estimated as

$$V_v = x\langle S_z^{\mathrm{Mn}}\rangle \frac{N_0\langle X|J_{\mathrm{ve}}(\mathbf{r})|X\rangle}{6\hbar\omega_{c0}}.$$

Here the exchange integral for the valence band $\langle X|J_{\mathrm{ve}}(\mathbf{r})|X\rangle$ is negative.

At the typical magnetic fields of the order of several tesla, the magnetic length $l_c=\sqrt{\hbar c/eB}\sim 10$ nm and the cyclotron energy $\hbar\omega_{c0}=\hbar^2/m_0 l_c^2\sim 10^{-3}$ eV, while $N_0\langle X|J_{\mathrm{ve}}(\mathbf{r})|X\rangle = -0.88$ eV.[16] Therefore, $V_v \gg 1$.

We again choose the gauge $\mathbf{A}=B(0,x,0)$ and introduce the operators $a, a^\dagger$ according to

$$k_x = -\frac{i}{\sqrt{2}l_c}(a-a^\dagger),\quad k_y = \frac{1}{\sqrt{2}l_c}(a+a^\dagger). \tag{42}$$

Replacing also the operators J_x, J_y with $J_\pm = J_x \pm iJ_y$ we get,

$$H_0 = -\hbar\omega_{c0}\left\{\left[\gamma_1 - \frac{5}{4}\gamma_2 + \gamma_2 J_z^2\right](a^\dagger a + 1/2) + \frac{\gamma_2}{4}(J_-^2 + J_+^2)[a^2 + (a^\dagger)^2] + \frac{\gamma_3}{4}(J_+^2 - J_-^2)[a^2 - (a^\dagger)^2] + \frac{e}{c}l_c^2\kappa\mathbf{JB}\right\}, \tag{43}$$

$$H(k_z) = -\hbar\omega_{c0}\left[aJ_+\left(J_z+\frac{1}{2}\right) - a^\dagger J_-(J_z - 1/2)\right]i\sqrt{2}\gamma_3(l_c k_z) - \hbar\omega_{c0}\frac{1}{2}\left(\gamma_1 + \frac{5}{2}\gamma_2 - 2\gamma_2 J_z^2\right)(l_c k_z)^2. \tag{44}$$

Further on we will use the spherical approximation, i.e., we set $\gamma_2=\gamma_3$. We get

$$-\frac{H}{\hbar\omega_{c0}} = 2g_v J_z + \left(\gamma_1 - \frac{5}{4}\gamma_2 + \gamma_2 J_z^2\right)(a^\dagger a + 1/2) + \frac{1}{2}\left(\gamma_1 + \frac{5}{2}\gamma_2 - 2\gamma_2 J_z^2\right)(l_c k_z)^2 + \frac{\gamma_2}{2}[J_+^2 a^2 + J_-^2(a^\dagger)^2] + [aJ_+(J_z+1/2) - a^\dagger J_-(J_z - 1/2)]i\sqrt{2}\gamma_2(l_c k_z), \tag{45}$$

where we have introduced the effective g factor in the valence band $g_v=\hbar\kappa/2+V_v$.

Due to large values of the exchange Hamiltonian g_v, we can omit the last two terms, i.e.,

$$V = [aJ_+(J_z+1/2) - a^\dagger J_-(J_z-1/2)]\sqrt{2}\gamma_2 l_c\frac{\partial}{\partial z} + \frac{\gamma_2}{2}[J_+^2 a^2 + J_-^2(a^\dagger)^2], \tag{46}$$

in Eq. (45) that sufficiently simplifies the problem. The reason of such a separation of the Hamiltonian is rather obvious, the Hamiltonian V leads to transitions changing both the spin and the Landau numbers and can be taken into account as a perturbation. In this approximation, the levels can be considered independently and we have, for the top heavy and light hole series of levels (in the hole representation),

$$E^{(\mathrm{hh})}_{-3/2,n,n_v} = E_g - 3\hbar\omega_{c0}g_v + \hbar\omega_{c0}\frac{m_0(3m_h+m_l)}{4m_l m_h}(n+1/2) + \frac{\pi^2\hbar^2 n_v^2}{2m_h L^2}, \tag{47}$$

$$\psi^{(\mathrm{hh})}_{-3/2} = \varphi_n(x - x_{0k_y})\chi_{n_v}(z)\frac{e^{ik_y y}}{\sqrt{L_y}}u_{-3/2}, \tag{48}$$

$$E^{(\mathrm{lh})}_{-1/2,n,n_v} = E_g - \hbar\omega_{c0}g_v + \hbar\omega_{c0}\frac{m_0(3m_l+m_h)}{4m_l m_h}(n+1/2) + \frac{\pi^2\hbar^2 n_v^2}{2m_l L^2}, \tag{49}$$

$$\psi^{(\mathrm{lh})}_{-1/2} = \varphi_n(x - x_{0k_y})\chi_{n_v}(z)\frac{e^{ik_y y}}{\sqrt{L_y}}u_{-1/2}. \tag{50}$$

Here E_g is the gap, $m_l(m_h)$ are the light (heavy) hole masses, n_v is the quantization number of transverse motion, and $\chi_{n_v}(z)$ is the corresponding wave function. We take into account that the γ_1, γ_2 parameters are related to the effective masses by $\gamma_1 = m_0(m_h+m_l)/2m_h m_l$ and $\gamma_2 = m_0(m_h - m_l)/4m_h m_l$.

In this zeroth approximation, phonons cannot induce transitions between these states. In the next approximation of perturbation theory with respect to V, these states are mixed and we get for the top heavy-hole state $\psi^{(\mathrm{hh})}_{-3/2,n,n_v}$,

$$\psi^{(\mathrm{hh})}_{-3/2,0,1} = \varphi_0(x - x_{0k_y})\chi_1(z)\frac{e^{ik_y y}}{\sqrt{L_y}}u_{-3/2}. \tag{51}$$

For the light-hole top state we have

$$\psi^{(\mathrm{lh})}_{-1/2,0,1} = \frac{e^{ik_y y}}{\sqrt{L_y}}\left[\varphi_0(x - x_{0k_y})\chi_1(z)u_{-1/2} + \frac{4}{3}\frac{\gamma_2\hbar\omega_{c0}(l_c/L)}{E_{-1/2,0,1} - E_{-3/2,1,2}}\varphi_1(x - x_{0k_y})\chi_2(z)u_{-3/2}\right]. \tag{52}$$

Now it is obvious that a phonon can induce transitions between these states. Suppose that by sweeping the magnetic field we can achieve the hole-phonon resonance condition between the states described by Eqs. (51) and (52),

$$E_{-1/2,0,1} - E_{-3/2,0,1} = \hbar\omega_{\mathrm{LO}},$$

or

$$2g_v\hbar\omega_{c0} - \frac{m_0(m_h - m_l)}{4m_h m_l}\left(\hbar\omega_{c0} - 2\frac{\pi^2\hbar^2}{m_0 L^2}\right) = \hbar\omega_{\mathrm{LO}}.$$

For the value $\Delta_{-1/2,-3/2}\equiv\Delta_v$ describing the splitting in the valence band, we get at the resonant condition,

$$\Delta_v^2 = 4\alpha\frac{l_{\mathrm{LO}}}{l_c}(\hbar\omega_{\mathrm{LO}})^2 6\left[\frac{m_0(m_h - m_l)}{4m_h m_l}\right]^2\left(\frac{\hbar\omega_{c0}}{\hbar\omega_{\mathrm{LO}}}\right)^2\left(\frac{l_c}{L}\right)^2 f_v(l_c/L),$$

where

$$f_v\left(\frac{l_c}{L}\right) = l_c \int dz_1 dz_2 \chi_1(z_1)\chi_2(z_1)\chi_1(z_2)\chi_2(z_2) \int_0^\infty dq_\perp \frac{q_\perp^2 l_c^2}{2} \exp(-l_c^2 q_\perp^2/2 - q_\perp |z_1 - z_2|),$$

$$f_v(x) = x, x \ll 1, f_v(x) = (10/9\pi^2 x), x \gg 1. \quad (53)$$

Let us compare the SMPR splittings in the conduction and valence bands. We evaluate

$$\frac{\Delta_{0+,0}}{\Delta_v} \sim \left(\frac{\alpha_R}{L\hbar^2/2m_0L^2}\right)\left(\frac{g_v}{g_c}\right)^{3/4}\left[\frac{4m_h m_l}{m_0(m_h - m_l)}\right] \times \left[\frac{f(l_c/L)}{f_v(2\sqrt{g_v/g_c}l_c/L)}\right]^{1/2},$$

and see that the splitting in the conduction band is bigger than in the valence band and is determined by the parameter g_v/g_c. Here l_c is the magnetic length for magnetic fields that is required to achieve the resonance condition in the conduction band.

In principle, in valence band one can also write the spin-orbital term of Rashba type,[23]

$$H_{\mathrm{vR}} = \frac{\alpha'}{\hbar}[\mathbf{J}p]\mathbf{n},$$

which in the magnetic field can be rewritten as

$$H_{vR} = \frac{\alpha'}{\sqrt{2}l_c}\{J_+ a + J_- a^+\}.$$

This term leads to the ratio

$$\frac{\Delta_{0+,0}}{\Delta_v} \sim \left(\frac{\alpha_R}{\alpha'}\right)\left(\frac{g_v}{g_c}\right)^{3/4}\left[\frac{f(l_c/L)}{f_v'(2\sqrt{g_v/g_c}l_c/L)}\right]^{1/2},$$

where

$$f_v'\left(\frac{l_c}{L}\right) = \int dz_1 dz_2 \chi_1^2(z_1)\chi_2^2(z_2) \int_0^\infty dq q^4 \times \exp(-q^2/2 - q|z_1 - z_2|/l_c). \quad (54)$$

Although the SMPR condition is met first for the hole states as one sweeps the magnetic field, the splitting in the valence band turns out to be much smaller than in the conduction band. This is the consequence of the smaller spin-phonon coupling strength for the states that are strongly shifted by the Zeeman energy.

V. RESONANT REFLECTION AND TRANSMISSION

We consider the simplest geometry where the wave $\sim e^{ikz}$ is perpendicularly incident to the plane of the well. Neglecting in the induced current the longitudinal part (this term in the induced current has a small factor $u_0 \simeq \alpha_R/\mu_B g_c B l_c$) so that we can put $\nabla\cdot\mathbf{D} = \epsilon_b \nabla\cdot\mathbf{E} = 0$ the Maxwell equation for the wave with frequency ω can be written as (in this section k denotes the wave vector of light)

$$\frac{d^2}{dz^2}E_\alpha + k^2 E_\alpha = \frac{4\pi}{\hbar c^2}\int dz' \Pi^R_{\alpha\beta}(z,z',\omega)E_\beta(z'). \quad (55)$$

Here $k^2 = \omega^2\epsilon_b/c^2$ (we neglect the difference in the background susceptibilities of the well and barriers). We have taken into account that the polarization operator $\Pi_{\alpha\beta}$ (here the averaging over the distances that are much greater than the lattice parameter is implied) is

$$\Pi^R(z,z',\omega) = \int dx' dy' \Pi^R(\mathbf{r},\mathbf{r}',\omega). \quad (56)$$

The Green's function of operator $d^2/dz^2 + k^2$ obeys the equation

$$\left(\frac{d^2}{dz^2} + k^2\right)\mathcal{G}(z,z') = -\delta(z - z'), \quad (57)$$

and is given by

$$\mathcal{G}^\pm(z,z') = \pm\frac{i}{2k}e^{\pm ik|z-z'|}. \quad (58)$$

For the transmission and reflection problem, one should use $\mathcal{G}^+(z,z')$ function. Then the solution of Eq. (55) can be written as

$$E_\alpha = E_\alpha^0 e^{ikz} - \frac{4\pi}{\hbar c^2}\int dz' dz'' \mathcal{G}^+(z,z')\Pi^R_{\alpha\beta}(z',z'',\omega)E_\beta(z''), \quad (59)$$

where E_α^0 is the amplitude of the incident wave. For $z > L$, where L is the width of the quantum well, we can identify the transmitted wave as

$$E_\alpha^t = E_\alpha^0 e^{ikz} - \frac{2i\pi}{k\hbar c^2}e^{ikz}\int_0^L dz' dz'' e^{-ikz'}\Pi^R_{\alpha\beta}(z',z'',\omega)E_\beta(z''), \quad (60)$$

and the reflected one can be identified considering $z < 0$,

$$E_\alpha^r = -\frac{2i\pi}{k\hbar c^2}e^{-ikz}\int_0^L dz' dz'' e^{ikz'}\Pi^R_{\alpha\beta}(z',z'',\omega)E_\beta(z''). \quad (61)$$

Assuming that $\Pi^R(z,z')$ can be factorized as $\Pi^R(z,z') = \Pi^{(1)}(z)\Pi^{(2)}(z')$ [such a factorization is possible since below we will consider transitions between two fixed states with respect to transverse motion $\chi_{n_v}(z)$ and $\chi_{n_c}(z)$], we scalarly multiply Eq. (59) by $\Pi_\alpha^{(2)}(z)$ and integrate over z, then we get

$$\mathcal{F} = -\mathcal{F}\frac{4\pi}{\hbar c^2}\int dz dz' \mathcal{G}^+(z,z')\Pi^R_{\alpha\alpha}(z,z',\omega) + E_\alpha^0\int_0^L dz e^{ikz}\Pi_\alpha^{(2)}(z), \quad (62)$$

where we have the introduced notation,

$$\mathcal{F} = \int_0^L dz' \Pi_\beta^{(2)}(z',\omega) E_\beta(z').$$

Solving Eq. (62) for $\mathcal{F}$ and making use of Eqs. (60) and (61), we get for the amplitudes of the transmitted and reflected waves,

$$E_\alpha^t = \left[\delta_{\alpha\beta} + \frac{4\pi \int dz' dz e^{-ik(z-z')} \Pi_{\alpha\beta}^R(z,z',\omega)}{2ikc^2\hbar - 4\pi \int dz' dz e^{ik|z-z'|} \Pi_{\gamma\gamma}^R(z,z',\omega)} \right] E_\beta^0, \tag{63}$$

$$E_\alpha^r = \frac{4\pi \int dz' dz e^{ik(z+z')} \Pi_{\alpha\beta}^R(z,z',\omega)}{2ikc^2\hbar - 4\pi \int dz' dz e^{ik|z-z'|} \Pi_{\gamma\gamma}^R(z,z',\omega)} E_\beta^0. \tag{64}$$

In the basis $\mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$ in our approximation, only one component of $\Pi_{\alpha\beta}$ is nonvanishing, i.e., $\Pi_{++} = 2\Pi_{xx}$. Due to the symmetry relations, we have $\Pi_{xx} = \Pi_{yy} = i\Pi_{xy} = -i\Pi_{yx}$. Therefore, left circularly polarized incident wave $\mathbf{e}_-$ is not reflected, while for the right polarized incident wave $\mathbf{e}_+$ we get,

$$t_+ = 1 + \frac{4\pi \int dz' dz e^{-ik(z-z')} \Pi_{++}^R(z,z',\omega)}{2ikc^2\hbar - 4\pi \int dz' dz e^{ik|z-z'|} \Pi_{++}^R(z,z',\omega)}, \tag{65}$$

for the transmission coefficient $\mathbf{E}^t = t_+ E_+^0 \mathbf{e}_+ e^{ikz}$ and

$$r_+ = \frac{4\pi \int dz' dz e^{ik(z+z')} \Pi_{++}^R(z,z',\omega)}{2ikc^2\hbar - 4\pi \int dz' dz e^{ik|z-z'|} \Pi_{++}^R(z,z',\omega)}, \tag{66}$$

for the reflection (amplitude) coefficient $\mathbf{E}^r = r_+ E_+^0 \mathbf{e}_+ e^{-ikz}$. Since the propagation direction of the wave is now inverted, the reflected wave has the left polarization. A linearly polarized incident wave will be reflected as a circularly left polarized wave. In the case where the wavelength $2\pi/k$ is bigger than the well width L (i.e., $kL \ll 1$), the exponential factors can be omitted.

Let us consider the polarization operator. We can write the formal expression for the operator,

$$\Pi_{\alpha\beta}^R(\mathbf{r},\mathbf{r}',\omega) = -\frac{i}{2} \sum_{\lambda_1\lambda_2} j_{\lambda_2\lambda_1}^\alpha(\mathbf{r}) j_{\lambda_1\lambda_2}^\beta(\mathbf{r}') \int \frac{d\varepsilon}{2\pi\hbar} \times \left\{ \tanh\frac{\varepsilon+\hbar\omega}{2T} [G_{\lambda_1}^R(\varepsilon/\hbar+\omega) - G_{\lambda_1}^A(\varepsilon/\hbar+\omega)] G_{\lambda_2}^A(\varepsilon/\hbar) + \tanh\frac{\varepsilon}{2T} [G_{\lambda_2}^R(\varepsilon/\hbar) - G_{\lambda_2}^A(\varepsilon/\hbar)] G_{\lambda_1}^R(\varepsilon/\hbar) \right\}, \tag{67}$$

where

$$\mathbf{j}_{\lambda_1\lambda_2}(\mathbf{r}) = \frac{ie\hbar}{2m_0} \{\Phi_{\lambda_1}^*(\mathbf{r}) \nabla \Phi_{\lambda_2}(\mathbf{r}) - [\nabla\Phi_{\lambda_1}^*(\mathbf{r})]\Phi_{\lambda_2}(\mathbf{r})\} - \frac{e^2}{m_0 c} \mathbf{A}_0 \Phi_{\lambda_1}^*(\mathbf{r}) \Phi_{\lambda_2}(\mathbf{r}). \tag{68}$$

Here $\Phi_\lambda(\mathbf{r})$ are the eigenfunctions of the Hamiltonian and $\mathbf{A}_0$ is the vector potential of the applied static magnetic field. We consider the interband transitions and assume that the valence-band states are occupied while the states in the conduction band are empty. Keeping only the resonant contribution in Eq. (67) we get,

$$\Pi_{\alpha\beta}^R(\mathbf{r},\mathbf{r}',\omega) = \sum_{\lambda_c\lambda_v} j_{\lambda_v\lambda_c}^\alpha(\mathbf{r}) j_{\lambda_c\lambda_v}^\beta(\mathbf{r}') \int \frac{d\varepsilon}{2\pi\hbar i} G_{\lambda_c}^R(\varepsilon/\hbar+\omega) G_{\lambda_v}^A(\varepsilon/\hbar). \tag{69}$$

We can consider the states in the valence band as unchanged by the electron-phonon interaction (since we are interested only in the splitting phenomenon in the conduction band) and for the circularly polarized wave we get,

$$r_+ = \frac{-i\Gamma(\hbar\omega - \varepsilon_1 - \hbar\omega_{\mathrm{LO}} + \varepsilon_v)}{(\hbar\omega - \varepsilon_+ + \varepsilon_v + i0)(\hbar\omega - \varepsilon_- + \varepsilon_v + i0)/\hbar + i\Gamma(\hbar\omega - \varepsilon_1 - \hbar\omega_{\mathrm{LO}} + \varepsilon_v)}, \tag{70}$$

where we have the introduced notation,

$$\Gamma = \frac{4\pi}{\hbar\omega c\sqrt{\varepsilon_b}} \sum_{\lambda_c\lambda_v} \int d\mathbf{r}' dz j_{\lambda_c\lambda_v}^x(\mathbf{r}') j_{\lambda_v\lambda_c}^x(\mathbf{r}). \tag{71}$$

This quantity can be related to the recombination rate of the transition under consideration. Here we have taken into account that $\Pi_{xx} = \Pi_{yy}$. Since we are interested in the splitting phenomenon, we assume that the resonance light frequency is close to the transition from the ground state in the + group

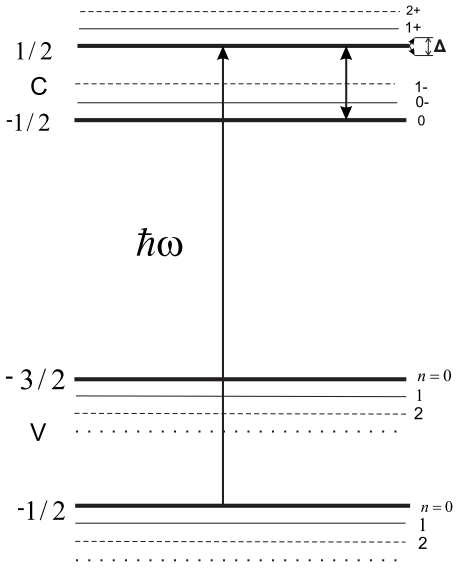

FIG. 3. Interband transitions.

in the conduction band to the ground state in the valence band with $J_z=-1/2$ (see Fig. 3).[24] Therefore, in the following formulae we set $\varepsilon_1=\varepsilon_{c1}=\varepsilon_0$, $\varepsilon_2=\varepsilon_{c2}=\varepsilon_{0+}$, $\varepsilon_{c2}-\varepsilon_{c1}=\mu_B g_c B$, $\varepsilon_v=-E^{(\mathrm{lh})}_{-1/2,0,1}$, and $\Delta=\Delta_{0+,0}$.

The specification of the transition between the states described by Eq. (50) and

$$\psi_{c0+}=S\chi_{n_c}(z)[\varphi_0(x-x_{0k_y})s_+ + u_0\varphi_1(x-x_{0k_y})s_-]\frac{e^{ik_y y}}{\sqrt{L_y}}, \tag{72}$$

allows us to express the recombination rate explicitly,

$$\Gamma=\frac{4\pi}{\hbar\omega c\sqrt{\epsilon_b}}\frac{e^2|p_{cv}|^2}{6m_0^2}\frac{1}{2\pi l_c^2}, \quad p_{cv}=\langle S|p_x|X\rangle. \tag{73}$$

Here we have assumed $|\langle\chi_{v1}|\chi_{c1}\rangle|^2=1$ for the overlapping of the transverse quantized wave function of the conduction and valence bands. Now we will introduce the following dimensionless variables: the deviation from the SMPR $(\varepsilon_{c2}-\varepsilon_{c1}-\hbar\omega_{\mathrm{LO}})/\Delta=\delta$, the optical frequency $2\hbar(\omega-\omega_0)/\Delta=x$ [$\omega_0=(\varepsilon_{c2}-\varepsilon_v)/\hbar$ being the interband resonance frequency], and the uncertainty in the level energy position $2\hbar\Gamma/\Delta=\gamma$. Then we can write the power reflection coefficient $R=|r_+|^2$ as

$$R=\frac{\gamma^2(x+2\delta)^2}{(x+\delta-\sqrt{1+\delta^2})^2(x+\delta+\sqrt{1+\delta^2})^2+\gamma^2(x+2\delta)^2}. \tag{74}$$

If the dimensionless deviation $\delta\gg 1$ (i.e., the deviation from the phonon resonance condition is much bigger than the splitting) using $\sqrt{1+\delta^2}\simeq\delta$, we see that the single-line structure is restored,

$$R=\frac{\gamma^2}{x^2+\gamma^2}. \tag{75}$$

In the case of exact electron-phonon resonance that can be achieved by sweeping the magnetic field, $\delta=0$ and we get,

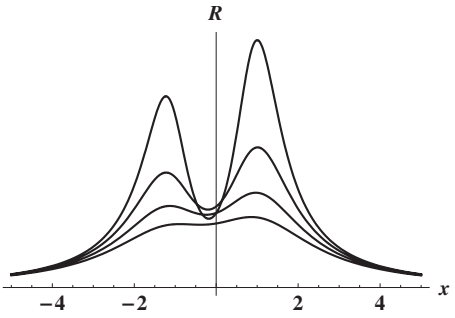

FIG. 4. Reflection coefficient as a function of optical frequency at $\delta=0.1$ for different level widths $\gamma_e=0.7,1,1.3,1.6$. Increase of γ_e results in the decrease and in the vanishing of the dip.

$$R=\frac{\gamma^2x^2}{(x^2-1)^2+\gamma^2x^2}. \tag{76}$$

In this case, the power reflection coefficients reach its maximal value under the optical resonant conditions. For linearly polarized incident wave, this maximal value is 1/2.

So far we have assumed that the energy uncertainty of the level under consideration is much smaller than the splitting Δ, otherwise the level splitting cannot be resolved. Indeed, we will have for the Green's function instead of Eq. (9),

$$G_2(\varepsilon)=\frac{1}{\varepsilon+i\hbar\Gamma_2-\varepsilon_2+\mu-(\Delta/2)^2/(\varepsilon+i\hbar\Gamma_1-\varepsilon_1-\hbar\omega_{\mathrm{LO}}+\mu)}, \tag{77}$$

provided that we take this uncertainty into account. Here we have phenomenologically introduced Γ_2 and Γ_1 for the corresponding energy levels ε_2 and ε_1, respectively. It is seen from this expression that even for $\varepsilon-\varepsilon_1-\hbar\omega_{\mathrm{LO}}=0$, we can discard the second term in the denominator since $\Delta\ll\hbar\Gamma_1$ and the level does not split. The recombination rate can be estimated by taking into account that $|p_{\mathrm{cv}}|^2/2m_0$ is of the order of the Bohr energy $\hbar\omega\sim E_g\sim 1.6$ eV. Then it is seen that $\gamma=\hbar\Gamma/\Delta\ll 1$.

Let us consider the case of equal $\Gamma_1=\Gamma_2$ widths of both levels. Then we can write for the reflection coefficient,

$$R=\frac{\gamma^2[(x+2\delta)^2+\gamma_e^2]}{(x+\delta-\sqrt{1+\delta^2+\gamma_e^2})^2(x+\delta+\sqrt{1+\delta^2+\gamma_e^2})^2+4\gamma_e^2(x+\delta)^2}, \tag{78}$$

where we introduce a dimensionless quantity proportional to the sum of level widths $\gamma_e=4\hbar\Gamma_1/\Delta$ and neglect the level width due to the recombination processes. Figure 4 demonstrates how the increasing of the level widths smears the doublet structure of the reflection line. The symmetry of this doublet structure depends on the deviation from the spin electron-phonon resonance (Fig. 5).

VI. APPLICABILITY OF PERTURBATION THEORY

In Sec. II we considered only the simplest diagram for the self-energy. Now we are going to discuss the validity of this approximation for the electron-phonon interaction in some

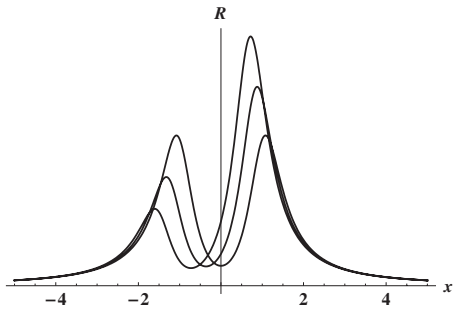

FIG. 5. Reflection coefficient for various values of deviation from the spin electron-phonon resonance $\delta=0,0.2,0.4$ at $\gamma_e=0.5$. The symmetric curve corresponds to $\delta=0$. Increase of the deviation results in an asymmetric line structure and eventually in a one-peak line structure.

detail. The point is that the lowest approximation of the perturbation theory used sometimes for the treatment of a two-dimensional (2D) case in the magnetic field is by no means always applicable. We believe that it is worthwhile to investigate the limits of applicability of this theory. It is easily seen that each additional phonon line in the higher-order diagrams can bring about an additional resonant denominator; therefore, we should consider the series of the most diverging sequence of diagrams. The situation is not unique and has been encountered earlier in the polaron problem in the three-dimensional case and such a consequence of diagrams was first considered by Pitaevskii.[25]

We consider two empty states 1 and 2 with energies $\varepsilon_{1,2}$ $\varepsilon_2=\varepsilon_1+\hbar\omega_{\mathrm{LO}}$. Each state is unoccupied $\varepsilon_{1,2}>\mu$. Therefore, we can write for the electron Green's function,

$$G(\varepsilon,\mathbf{r}_1,\mathbf{r}_2)=\sum_{\nu=1}^{2}\frac{\Psi_\nu(\mathbf{r}_1)\Psi^*(\mathbf{r}_2)}{\varepsilon-(\varepsilon_\nu-\mu)+i0}. \tag{79}$$

The phonon Green's function can be written as

$$D(\omega,\mathbf{r}_1,\mathbf{r}_2)=\sum_{\alpha\mathbf{q}}|C_{\alpha,\mathbf{q}}|^2\left(\frac{e^{-i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}}{\omega-\hbar\omega_{\mathrm{LO}}+i0}-\frac{e^{i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)}}{\omega+\hbar\omega_{\mathrm{LO}}-i0}\right), \tag{80}$$

where $|C_\alpha|^2\to 2\pi e^2\hbar\omega_{\mathrm{LO}}/q^2\epsilon^*$. We are to evaluate the Green's function for the state 2. Since we consider the empty electron states above the chemical potential, the self-energy

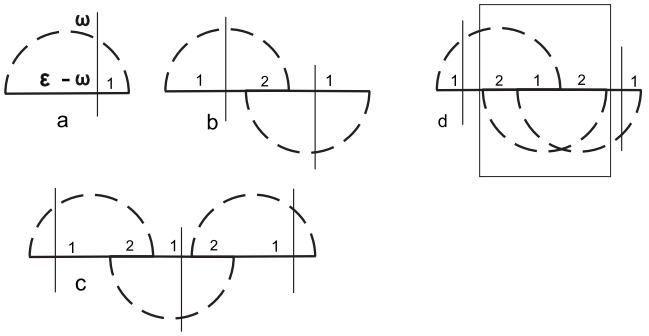

FIG. 6. Self-energy diagrams. Resonant sections are shown by vertical lines.

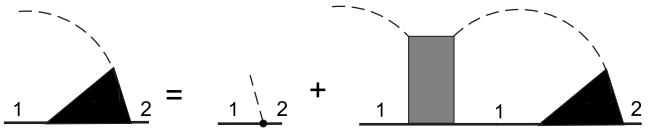

FIG. 7. Equation for the vertex.

diagrams will involve Green's functions of the type,

$$\frac{1}{\varepsilon-\omega-(\varepsilon_{1,2}-\mu)+i0}.$$

These functions have the pole with respect to ω in the upper half plane. Therefore, we keep—in the phonon Green's function—only the part having the pole with respect to ω in the lower half plane (otherwise the integration over ω vanishes), i.e.,

$$\frac{1}{\omega-\omega_{\mathrm{LO}}+i0}.$$

The simplest electron self-energy diagram [see the diagram (a) in Fig. 6] has a resonant denominator,

$$\Sigma_2(\varepsilon)\sim i\int\frac{d\omega}{2\pi}G(\varepsilon-\omega)D(\omega)=\frac{1}{\varepsilon-\omega_{\mathrm{LO}}-(\varepsilon_1-\mu)+i0}, \tag{81}$$

when ε is in the vicinity of $\varepsilon_2=\varepsilon_1+\omega_{\mathrm{LO}}$. Diagrams with more resonances are of two types; the first type leads to corrections to the Green's function [to the line 1 in the skeleton diagram (a) in Fig. 6] and they can be taken into account regarding the Green's function as renormalized, the second type leads to the corrections to the electron-phonon vertex. Since the corrections of the first type can be taken into account perturbatively (these diagrams do not involve resonant denominators), we will not consider them and concentrate on the diagrams of the second type. Several diagrams of the last type are presented in Fig. 6. The diagrams (b) and (c) involve two and three resonant denominators, respectively. We can draw more complicated diagrams with two resonance denominators [similar to diagram (d) in Fig. 6]; it is now seen that the diagrams of this type can be regarded as the diagram (b) with a block that does not involve resonant denominators, we will call such a block a compact block. Therefore, we can write the integral equation for the renormalized vertex (see Fig. 7). In Fig. 8 we show that the compact block is the expansion with respect to the electron-phonon coupling strength; therefore, we write the integral equation keeping only the first term in this expansion,

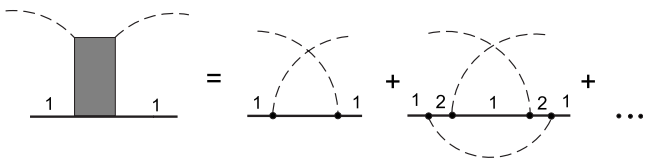

FIG. 8. Block expansion.

$$\Gamma(\varepsilon-\omega,k_y-q_y,\varepsilon,k_y,q_x,q_z)=\langle 1k_y-q_y|e^{-iq_xx-iq_zz}|2k_y\rangle, \tag{82}$$

$$+i\int\frac{d\omega' d\mathbf{q}'}{(2\pi)^4}\langle 1k_y-q_y|e^{iq_x'x+iq_z'z}|2k_y-q_y-q_y'\rangle\langle 2k_y-q_y-q_y'|e^{-iq_xx-iq_zz}|1k_y-q_y'\rangle, \tag{83}$$

$$\times\frac{|C_{\mathbf{q}'}|^2\Gamma(\varepsilon-\omega',k_y-q_y',\varepsilon,k_y,q_x',q_z')}{(\omega'-\omega_{\mathrm{LO}}+i0)[\varepsilon-\omega-\omega'-(\varepsilon_2-\mu)+i0][\varepsilon-\omega'-(\varepsilon_1-\mu)+i0]}. \tag{84}$$

Let us write this equation for the specific states $1=\{n=0,i=1\}$ and $2=\{n=0+,i=1\}$ (see Sec. III), i.e., we consider the ground states with respect to the orbital motion and to the spatial confinement. In order to simplify the integral equation we introduce the function $A(q_\perp,\varepsilon,\omega)$ by relation,

$$\Gamma(\varepsilon-\omega,k_y-q_y,\varepsilon,k_y,q_x,q_z)$$
$$=e^{iq_x(k_y-q_y/2)}e^{-q_\perp^2/4}\frac{q_y-iq_x}{\sqrt{2}}\langle 1|e^{-iq_zz}|1\rangle A(q_\perp,\varepsilon,\omega), \tag{85}$$

where $q_\perp=\sqrt{q_x^2+q_y^2}$ and the wave vectors are dimensionless (the factor is the magnetic length). Then, using the relation for the phase factors under the integral,

$$e^{i(q_yq_x'-q_xq_y')}=\sum_{n=-\infty}^{\infty}J_2(q_\perp q_\perp)e^{-in(\varphi'-\varphi)}, \tag{86}$$

we get,

$$A(q,\varepsilon,\omega)=1+\frac{i}{2}\int\frac{d\omega' p^3dp}{(2\pi)^3}e^{-p^2/2}\frac{J_2(qp)\phi(p)A(p,\varepsilon,\omega')}{(\omega'-\omega_{\mathrm{LO}}+i0)[\varepsilon-\omega-\omega'-(\varepsilon_2-\mu)+i0][\varepsilon-\omega'-(\varepsilon_1-\mu)+i0]}, \tag{87}$$

$$\phi(p)=\frac{u_0^2}{l_c}\int dq_z|C_{p,q_z}|^2|\langle 1|e^{iq_zz/l_c}|1\rangle|^2. \tag{88}$$

Now we suppose that the function $A(q,\varepsilon,\omega)$ has no poles with respect to ω in the lower half complex plane and consider the case when the magnetic length is much bigger than the quantum well width. The last assumption leads to $\phi(p)=\pi/pl_c$ and we can rewrite the integral equation for $A(q,\varepsilon,\omega_{\mathrm{LO}})\equiv A(q,\varepsilon)$ as the Fredholm equation,

$$A(q,\varepsilon)=1+\lambda\int_0^\infty dpp^2e^{-p^2/2}J_2(qp)A(p,\varepsilon), \tag{89}$$

where parameter λ includes the resonant denominator,

$$\lambda=-\frac{1}{8}\frac{u_0^2(e^2/\varepsilon^*l_c)}{\varepsilon-(\varepsilon_1+\hbar\omega_{\mathrm{LO}}-\mu)+i0}. \tag{90}$$

In reality the uncertainty of the level $i\hbar/\tau$ enters the last equation instead of $i0$. Let us evaluate the minimal value of τ, where we remain within the framework of perturbation theory and it is then sufficient to consider only the skeleton diagram for the self-energy. With $\hbar\omega_{\mathrm{LO}}=0.02$ eV, $B=3$ T, $m_c=0.1m_0$, $\alpha_R=10^{-9}$ eV cm, and $\alpha=0.39$, we get that the perturbation scheme is valid for the relaxation times shorter than $\tau_0=5\times10^{-10}$ sec. On the other hand, to resolve the splitting, the level uncertainty must be smaller than the level splitting $\Delta\simeq5\times10^{-4}$ eV. This requires the times to be bigger than 10^{-12} sec. Therefore, there exists a region of relaxation times $\sim10^{-11}$ sec, where the perturbation theory is valid and the splitting phenomena is discernable.

Here we wish to emphasize that in an ordinary situation of the magnetophonon phenomena, one encounters the case where the integral term in Eq. (87) cannot be discarded and an integral equation should be solved. This could lead to bound states in the spectrum. Therefore, we conclude that the approaches, taking into account only the one phonon processes described by the skeleton self-energy diagram, cannot be considered as reliable.

VII. CONCLUSIVE REMARKS

We have considered optical manifestation of SMPR in semimagnetic semiconductors. Due to the electron-phonon coupling, the resonant reflection and transmission line representing the interband transitions is split into two lines. The distance between the lines is determined by the strength of the electron-phonon coupling.

We should, however, indicate that some points have not been taken into account in our calculation. Among them the most important is the natural width of the phonon levels. For the optical phonons at low temperatures, it is determined by the decay of an optical phonon into two acoustic ones.

The natural width of the electron lines is also important. It may be determined by the collisions of electrons with acoustic phonons and with the defects of the lattice, as well as by recombination. These effects result in the widening of the

lines that has been briefly discussed. Under the conditions where these effects are strong, the lines may overlap as has been indicated above.

So far we have considered a situation where the equilibrium concentration of the carriers is so low that they do not influence the light absorption. One can conceive, however, another case of interest where, for instance, in equilibrium electrons (provided by donors outside the well) fill the conduction band up to the Fermi level. In such a case, transitions between the valence band and the states of the conduction band above the Fermi level are allowed. The oscillator strength for these transitions may be bigger than for those treated in this paper. One can expect that the width of the electron level in the conduction band should be rather small since the electrons can emit acoustic phonons with the energies not bigger than the spacing between the level they occupy and the Fermi level. However, one can expect that the width of the level in the valence band may be much bigger. Indeed, the holes can emit phonons with comparatively large energies as the spacing between their level and the top of the valence band can be rather large.

Experimental observation of SMPR can provide information about the electron-phonon interaction. Its investigation can also provide important information concerning various contributions into spin-orbit interaction as well as the strength of the exchange interaction.

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